

1.- Calcula:

$$a) (3\sqrt{2} + 2\sqrt{3})(\sqrt{2} + \sqrt{3}) = 3\sqrt{2}\sqrt{2} + 3\sqrt{2}\sqrt{3} + 2\sqrt{3}\sqrt{2} + 2\sqrt{3}\sqrt{3} = 12 + 5\sqrt{6}$$

$$b) (\sqrt{2} + 1)(\sqrt{3} - 2) = \sqrt{6} - 2\sqrt{2} + \sqrt{3} - 2$$

$$c) (\sqrt{3} - 2)(4 + \sqrt{3}) = 4\sqrt{3} + 3 - 8 - 2\sqrt{3} = 2\sqrt{3} - 5$$

$$d) (3 - 2\sqrt{2})(3 + 2\sqrt{2}) = 9 + 6\sqrt{2} - 6\sqrt{2} - 8 = 1$$

$$e) (1 - \sqrt{2})(1 + \sqrt{2}) = 1 - (\sqrt{2})^2 = -1$$

f)

$$(\sqrt{3} + 2\sqrt{2})(2\sqrt{3} + \sqrt{5}) = 2(\sqrt{3})^2 + \sqrt{3}\sqrt{5} + 4\sqrt{2}\sqrt{3} + 2\sqrt{2}\sqrt{5} = 6 + \sqrt{15} + 4\sqrt{6} + 2\sqrt{10}$$

2.- Calcula:

$$a) (2\sqrt{2} - 3\sqrt{2})^2 = 4(\sqrt{2})^2 - 12(\sqrt{2})^2 + 9(\sqrt{2})^2 = 8 - 24 + 18 = 2 \text{ (ver apdo. c)}$$

$$b) (1 - \sqrt{2})^2 = 1 - 2\sqrt{2} + (\sqrt{2})^2 = 3 - 2\sqrt{2}$$

$$c) (4\sqrt{2} + 3\sqrt{2})^2 = (7\sqrt{2})^2 = 49 \cdot 2 = 98 \text{ (el apartado a se puede hacer así)}$$

$$d) (\sqrt{2} + 1)^2 - (2\sqrt{2} + 3)^2 = 2 + 2\sqrt{2} + 1 - (8 + 12\sqrt{2} + 9) = -14 - 10\sqrt{2}$$

$$e) 2(\sqrt{2} - 3)^2 + (\sqrt{2} - \sqrt{3})^2 = 2(2 - 6\sqrt{2} + 9) + 2 - 2\sqrt{6} + 3 = 27 - 12\sqrt{2} - 2\sqrt{6}$$

$$f) \sqrt{2}(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3}) = \sqrt{2}[(\sqrt{2})^2 - (\sqrt{3})^2] = -\sqrt{2}$$

$$g) (\sqrt{3} - 2)^2 - (2\sqrt{2} - 4)^2 = 3 - 4\sqrt{3} + 4 - (8 - 16\sqrt{2} + 16) = -17 + 16\sqrt{2} - 4\sqrt{3}$$

$$h) (2\sqrt{2} + 3)^2 - (3\sqrt{2} + 1)^2 = 8 + 12\sqrt{2} + 9 - (18 + 6\sqrt{2} + 1) = -2 + 6\sqrt{2}$$

$$i) (\sqrt{3} - 1)^2 + (\sqrt{3} + 2\sqrt{2})^2 = 3 - 2\sqrt{3} + 1 + 3 + 4\sqrt{6} + 8 = 15 - 2\sqrt{3} + 4\sqrt{6}$$

3.- Calcula:

a) $(\sqrt{3})^4 = \sqrt{3^4} = 3^2 = 9$

b) $(\sqrt[4]{a})^5 = \sqrt[4]{a^5} = \sqrt[4]{a^4 a} = \sqrt[4]{a} \sqrt[4]{a^4} = a \sqrt[4]{a}$

c) $(\sqrt[5]{3a})^7 = \sqrt[5]{(3a)^7} = 3a \sqrt[5]{3^2 a^2} = 3a \sqrt[5]{9a^2}$

d) $(\sqrt[7]{2a^5})^6 = \sqrt[7]{2^6 a^{30}} = \sqrt[7]{64a^{28} a^2} = a^4 \sqrt[7]{64a^2}$

e) $(\sqrt[3]{3a^2})^6 = \sqrt[3]{3^6 a^{12}} = 3^2 a^4 = 9a^4$

f) $(\sqrt[4]{2ab^2c^3})^5 = \sqrt[4]{2^5 a^5 b^{10} c^{15}} = 2ab^2c^3 \sqrt[4]{2ab^2c^3}$

4.- Expresa como un solo radical:

a) $\sqrt{3\sqrt{4}} = \sqrt{\sqrt{3^2 \cdot 4}} = \sqrt[4]{36} = \sqrt[4]{6^2} = \sqrt{6}$

b) $\sqrt[6]{\sqrt{2}} = \sqrt[12]{2}$

c) $\sqrt[3]{\sqrt[5]{3a}} = \sqrt[30]{3a}$

d) $\sqrt{2\sqrt[3]{3}} = \sqrt{\sqrt[3]{2^3 \cdot 3}} = \sqrt[6]{24}$

e) $\sqrt{2\sqrt[3]{2^2} \sqrt{2}} = \sqrt{2\sqrt[3]{2^4 \cdot 2}} = \sqrt{2\sqrt[3]{2^5}} = \sqrt[6]{2^6 \cdot 2^5} = \sqrt[12]{2^{11}}$

f) $\sqrt[3]{3\sqrt[3]{9\sqrt[4]{3}}} = \sqrt[3]{3\sqrt[3]{4\sqrt[4]{9^4 \cdot 3}}} = \sqrt[3]{3\sqrt[3]{3^3 \sqrt[4]{3^9}}} = \sqrt[3]{\sqrt[3]{4\sqrt[3]{3^{12} \cdot 3^9}}} = \sqrt[36]{3^{21}} = \sqrt[12]{3^7}$

5.- Introduce factores en los siguientes radicales:

a) $2\sqrt{3} = \sqrt{2^2 \cdot 3} = \sqrt{12}$

b) $a^2 \sqrt[5]{b} = \sqrt[5]{a^{10} b}$

c) $2a \sqrt[3]{b} = \sqrt[3]{2^3 a^3 b} = \sqrt[3]{8a^3 b}$

d) $3\sqrt[4]{a^3 b} = \sqrt[4]{3^4 a^3 b} = \sqrt[4]{81a^3 b}$

e) $2a^3 \sqrt[4]{b^3} = \sqrt[4]{16a^{12} b^3}$

f) $5x^2 \sqrt[7]{y^4} = \sqrt[7]{5^7 x^{14} y^4}$