

$$53. \frac{\operatorname{sen} \alpha + \cos \alpha}{\cos \alpha} - \frac{\operatorname{sen} \alpha - \cos \alpha}{\operatorname{sen} \alpha} = \operatorname{sec} \alpha \cdot \operatorname{cosec} \alpha$$

$$\begin{aligned} & \frac{\operatorname{sen} \alpha + \cos \alpha}{\cos \alpha} - \frac{\operatorname{sen} \alpha - \cos \alpha}{\operatorname{sen} \alpha} = \frac{\operatorname{sen} \alpha (\operatorname{sen} \alpha + \cos \alpha) - \cos \alpha (\operatorname{sen} \alpha - \cos \alpha)}{\cos \alpha \operatorname{sen} \alpha} \\ & = \frac{\operatorname{sen}^2 \alpha + \cancel{\operatorname{sen} \alpha \cos \alpha} - \cancel{\operatorname{sen} \alpha \cos \alpha} + \cos^2 \alpha}{\cos \alpha \operatorname{sen} \alpha} = \frac{1}{\cos \alpha \operatorname{sen} \alpha} = \operatorname{sec} \alpha \cdot \operatorname{cosec} \alpha \end{aligned}$$

$$54. \frac{\cos^2 \alpha - \operatorname{sen}^2 \alpha}{1 - \operatorname{tg}^2 \alpha} = \cos^2 \alpha$$

$$\begin{aligned} & \frac{\cos^2 \alpha - \operatorname{sen}^2 \alpha}{1 - \operatorname{tg}^2 \alpha} = \frac{\cos^2 \alpha - \operatorname{sen}^2 \alpha}{1 - \frac{\operatorname{sen}^2 \alpha}{\cos^2 \alpha}} = \frac{\cos^2 \alpha - \operatorname{sen}^2 \alpha}{\frac{\cos^2 \alpha - \operatorname{sen}^2 \alpha}{\cos^2 \alpha}} = \\ & = \frac{\cos^2 \alpha (\cancel{\cos^2 \alpha - \operatorname{sen}^2 \alpha})}{\cancel{\cos^2 \alpha - \operatorname{sen}^2 \alpha}} = \cos^2 \alpha \end{aligned}$$

$$55. \frac{\cos \alpha + \operatorname{sen} \alpha - \operatorname{sen}^3 \alpha}{\operatorname{sen} \alpha} = \operatorname{cotg} \alpha + \cos^2 \alpha$$

$$\begin{aligned} & \frac{\cos \alpha + \operatorname{sen} \alpha - \operatorname{sen}^3 \alpha}{\operatorname{sen} \alpha} = \frac{\cos \alpha + \operatorname{sen} \alpha (1 - \operatorname{sen}^2 \alpha)}{\operatorname{sen} \alpha} = \\ & = \frac{\cos \alpha}{\operatorname{sen} \alpha} + \frac{\operatorname{sen} \alpha \cdot \cos^2 \alpha}{\operatorname{sen} \alpha} = \operatorname{cotg} \alpha + \cos^2 \alpha \end{aligned}$$

$$59. \frac{(2 \cos^2 \alpha - 1)^2}{\cos^4 \alpha - \operatorname{sen}^4 \alpha} = 1 - 2 \operatorname{sen}^2 \alpha$$

$$\begin{aligned} & \frac{[2 \cos^2 \alpha - (\operatorname{sen}^2 \alpha + \cos^2 \alpha)]^2}{(\cos^2 \alpha + \operatorname{sen}^2 \alpha)(\cos^2 \alpha - \operatorname{sen}^2 \alpha)} = \frac{(2 \cos^2 \alpha - \operatorname{sen}^2 \alpha - \cos^2 \alpha)^2}{\cos^2 \alpha - \operatorname{sen}^2 \alpha} = \\ & = \frac{(\cos^2 \alpha - \operatorname{sen}^2 \alpha)^2}{\cancel{\cos^2 \alpha - \operatorname{sen}^2 \alpha}} = 1 - \operatorname{sen}^2 \alpha - \operatorname{sen}^2 \alpha = 1 - 2 \operatorname{sen}^2 \alpha \end{aligned}$$