

Ejercicios resueltos

13.- $\text{sen}^2 \alpha (1 + \text{cotg}^2 \alpha) = 1$

(recuerda: $1 + \text{cotg}^2 \alpha = \text{cosec}^2 \alpha$)

$$\text{sen}^2 \alpha \cdot \text{cosec}^2 \alpha = \text{sen}^2 \alpha \cdot \frac{1}{\text{sen}^2 \alpha} = 1$$

14.- $(1 - \text{sen}^2 \alpha) (1 + \text{tg}^2 \alpha) = 1$

(recuerda: $1 + \text{tg}^2 \alpha = \text{sec}^2 \alpha$)

$$(1 - \text{sen}^2 \alpha) (1 + \text{tg}^2 \alpha) = \text{cos}^2 \alpha \cdot \text{sec}^2 \alpha = \text{cos}^2 \alpha \cdot \frac{1}{\text{cos}^2 \alpha} = 1$$

15.- $(\text{sen} \alpha + \text{cos} \alpha)^2 + (\text{sen} \alpha - \text{cos} \alpha)^2 = 2$

$$\begin{aligned} \text{sen}^2 \alpha + 2 \text{sen} \alpha \text{cos} \alpha + \text{cos}^2 \alpha + \text{sen}^2 \alpha - 2 \text{sen} \alpha \text{cos} \alpha + \text{cos}^2 \alpha &= \\ = 2 \text{sen}^2 \alpha + 2 \text{cos}^2 \alpha &= 2 (\text{sen}^2 \alpha + \text{cos}^2 \alpha) = 2 \end{aligned}$$

16.- $\text{tg}^2 \alpha \text{cos}^2 \alpha + \text{cot}^2 \alpha \text{sen}^2 \alpha = 1$

$$\frac{\text{sen}^2 \alpha}{\text{cos}^2 \alpha} \text{cos}^2 \alpha + \frac{\text{cos}^2 \alpha}{\text{sen}^2 \alpha} \text{sen}^2 \alpha = \text{sen}^2 \alpha + \text{cos}^2 \alpha = 1$$

17.- $\text{sec}^4 \alpha - \text{sec}^2 \alpha = \text{tg}^4 \alpha + \text{tg}^2 \alpha$

$$\begin{aligned} (1 + \text{tg}^2 \alpha)^2 - (1 + \text{tg}^2 \alpha) &= 1 + 2 \text{tg}^2 \alpha + \text{tg}^4 \alpha - 1 - \text{tg}^2 \alpha = \\ = \text{tg}^4 \alpha + \text{tg}^2 \alpha \end{aligned}$$

18.- $\text{cosec}^4 \alpha - \text{cosec}^2 \alpha = \text{cotg}^4 \alpha + \text{cotg}^2 \alpha$

$$\begin{aligned} (1 + \text{cotg}^2 \alpha)^2 - (1 + \text{cotg}^2 \alpha) &= 1 + 2 \text{cotg}^2 \alpha + \text{cotg}^4 \alpha - 1 - \text{cotg}^2 \alpha = \\ = \text{cotg}^4 \alpha + \text{cotg}^2 \alpha \end{aligned}$$

19.- $\text{sec} \alpha - \text{tg} \alpha = \frac{\text{cos} \alpha}{1 + \text{sen} \alpha}$

$$\begin{aligned} \frac{1}{\text{cos} \alpha} - \frac{\text{sen} \alpha}{\text{cos} \alpha} &= \frac{1 - \text{sen} \alpha}{\text{cos} \alpha} = \frac{1 - \text{sen} \alpha}{\text{cos} \alpha} \cdot \frac{1 + \text{sen} \alpha}{1 + \text{sen} \alpha} = \frac{(1 - \text{sen} \alpha) \cdot (1 + \text{sen} \alpha)}{\text{cos} \alpha \cdot (1 + \text{sen} \alpha)} = \\ = \frac{1 - \text{sen}^2 \alpha}{\text{cos} \alpha \cdot (1 + \text{sen} \alpha)} &= \frac{\text{cos}^2 \alpha}{\text{cos} \alpha \cdot (1 + \text{sen} \alpha)} = \frac{\text{cos} \alpha}{1 + \text{sen} \alpha} \end{aligned}$$

20.- $\text{cosec} \alpha - \text{cotg} \alpha = \frac{\text{sen} \alpha}{1 + \text{cos} \alpha}$

$$\frac{1}{\text{sen} \alpha} - \frac{\text{cos} \alpha}{\text{sen} \alpha} = \frac{1 - \text{cos} \alpha}{\text{sen} \alpha} \cdot \frac{1 + \text{cos} \alpha}{1 + \text{cos} \alpha} = \frac{\text{sen}^2 \alpha}{\text{sen} \alpha \cdot (1 + \text{cos} \alpha)} = \frac{\text{sen} \alpha}{1 + \text{cos} \alpha}$$